3rd Semester B.Tech Examination, Nov. 2004

MATHEMATICS-III

Full Marks : 70

Time : 3 hours

Answer **six** questions in all including **Q. No. 1** which is compulsory

The figures in the right-hand margin indicate marks

1. Answer the following questions : 2x10

(*i*) Solve the partial differential equation $u_{xy} = u_x$.

(*ii*) Test whether the partial differential equation

$$4u_{xx} - u_{yy}$$

is parabolic, elliptic or hyperbolic.

(*iii*) If $u(x, t) = \frac{1}{2}[f(x+ct) + f(x-ct)]$ satisfies the boundary condition $u(0, t) = u(1, t) = 0$, show that f must be an odd function with period 2.

(*iv*) Show that the only solution of $\nabla^2 u = 0$ depending only on $r = \sqrt{x^2 + y^2}$ is $u = a \ln r + b$ (Use the Laplacian in polar coordinates).

(*Turn Over*)

(*v*) Find the principal value of the argument of the complex numbers $z=-1$ and $z=i$.

(*vi*) Determine the nature of the curve (in the complex plane) given by the equation

$$\left|\frac{z-1}{z+1}\right|=1$$

(*vii*) Find all values of $\ln(-1)$

(*viii*) Determine the region of the complex plane given by the equation

$$|z-2-3i|<1$$

(*ix*) Find the fixed points of $w=f(z)=\dfrac{z-1}{z+1}$

(*x*) Find the value of

$$\int_C \frac{z^3+z^2+z+1}{z}\,dz$$

where C is the unit circle around the origin.

2. (*a*) Verify that $u(x, y)=a\ln(x^2+y^2)+b$ satisfies the Laplace equation $\nabla^2 u=0$. Determine a and b so that u satisfies the boundary condition $u(x,y)=0$ on the boundary of the circle $x^2+y^2=1$ and $u(x,y)=3$ on the circle $x^2+y^2=4$. 5

(*b*) Solve the equation by separation of variables:

$$xu_{xy}+2yu=0$$ 5

3. (*a*) Find $u(x, t)$ of the string of length $L=\pi$, $c^2=1$, initial velocity $=0$, and initial deflection

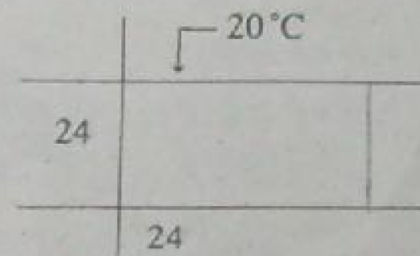
$$f(x)=\frac{x}{10}(\pi^2-x^2)$$ 5

(*b*) Solve the following partial differential equation using Laplace transforms

$$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=xt$$

$u(x, 0)=0$ if $x\geq 0$ and $u(0, t)=0$ if $t\geq 0$. 5

4. The face of a thin square copper plate are perfectly insulated. The upperside is kept at 20 °C. Find the steady-state temperature in the plate. 10



5. (*a*) Use $x=r\cos\theta$ and $y=r\sin\theta$ to transform

$$\nabla^2 u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$$

into its polar form. 5

(b) Find a linear fractional transformation which maps 1, i, -1 onto i, -1, $-i$ respectively. 5

6. (a) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic. Find the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$. 5

(b) Determine the Taylor's expansions of $f(z) = \frac{1}{1-z}$ about the point $a = 2$, and $a = 3$. Determine the radius of convergence in each case. 5

7. (a) Find the Laurent series expansion of $\frac{1}{z^2 - 3z + 2}$ valid in the region $1 < |z| < 2$. 5

(b) Evaluate the following integral by using the residue theorem

$$\int_0^{2\pi} \frac{1}{5 + 4\cos\theta}\, d\theta$$ 5

8. Evaluate the following integrals by using the residue theorem : 5+5

(a) $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 5x^2 + 4}$

(b) $\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx$

———