

3rd Semester B.Tech Examination, Nov. 2004

MATHEMATICS-III

Full Marks : 70

Time : 3 hours

Answer six questions in all including
Q. No. 1 which is compulsory

The figures in the right-hand margin indicate marks

1. Answer the following questions: 2 × 10

(i) Solve the partial differential equation

$$u_{xy} = u_x.$$

(ii) Test whether the partial differential equation

$$4u_{xx} - u_{yy}$$

is parabolic, elliptic or hyperbolic.

(iii) If $u(x, t) = \frac{1}{2} [f(x+ct) + f(x-ct)]$

satisfies the boundary condition

$$u(0, t) = u(1, t) = 0,$$

show that f must be an odd function with period 2.

(iv) Show that the only solution of $\nabla^2 u = 0$

depending only on $r = \sqrt{x^2 + y^2}$ is

$$u = a \ln r + b \quad (\text{Use the Laplacian in polar coordinates}).$$

(Turn Over)

(2)

(v) Find the principal value of the argument of the complex numbers $z = -1$ and $z = i$.

(vi) Determine the nature of the curve (in the complex plane) given by the equation

$$\left| \frac{z-1}{z+1} \right| = 1$$

(vii) Find all values of $\ln(-1)$

(viii) Determine the region of the complex plane given by the equation

$$|z - 2 - 3i| < 1$$

(ix) Find the fixed points of $w = f(z) = \frac{z-1}{z+1}$

(x) Find the value of

$$\int_C \frac{z^3 + z^2 + z + 1}{z} dz$$

where C is the unit circle around the origin.

2. (a) Verify that $u(x, y) = a \ln(x^2 + y^2) + b$ satisfies the Laplace equation $\nabla^2 u = 0$. Determine a and b so that u satisfies the boundary condition $u(x, y) = 0$ on the boundary of the circle $x^2 + y^2 = 1$ and $u(x, y) = 3$ on the circle $x^2 + y^2 = 4$. 5

(b) Solve the equation by separation of variables:

$$xu_{xy} + 2yu = 0 \quad 5$$

(3)

3. (a) Find $u(x, t)$ of the string of length $L = \pi$, $c^2 = 1$, initial velocity = 0, and initial deflection

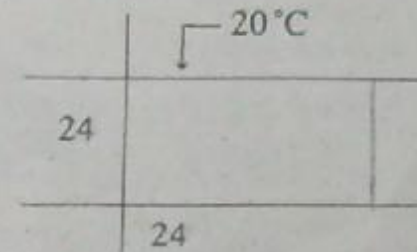
$$f(x) = \frac{x}{10} (\pi^2 - x^2) \quad 5$$

(b) Solve the following partial differential equation using Laplace transforms

$$x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = xt$$

$$u(x, 0) = 0 \text{ if } x \geq 0 \text{ and } u(0, t) = 0 \text{ if } t \geq 0. \quad 5$$

4. The face of a thin square copper plate are perfectly insulated. The upperside is kept at 20°C . Find the steady-state temperature in the plate. 10



5. (a) Use $x = r \cos \theta$ and $y = r \sin \theta$ to transform

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

into its polar form. 5

(b) Find a linear fractional transformation which maps $1, i, -1$ onto $i, -1, -i$ respectively. 5

6. (a) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic. Find the corresponding analytic function $f(z) = u(x, y) + iu(x, y)$. 5

(b) Determine the Taylor's expansions of $f(z) = \frac{1}{1-z}$ about the point $a=2$, and $a=3$. Determine the radius of convergence in each case. 5

7. (a) Find the Laurent series expansion of $\frac{1}{z^2 - 3z + 2}$ valid in the region $1 < |z| < 2$. 5

(b) Evaluate the following integral by using the residue theorem

$$\int_0^{2\pi} \frac{1}{5 + 4 \cos \theta} d\theta \quad 5$$

8. Evaluate the following integrals by using the residue theorem: 5+5

(a) $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 5x^2 + 4}$

(b) $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx$