IWL

2ND SEMESTER EXAMINATION, APRIL–2005

MATHEMATICS–II

Full Marks – 70

Time – 3 Hours

The figures in the right hand margin indicate full marks for the questions.

Answer Questions No. 1 which is compulsory and any ***five*** *from the rest.*

1. Answers to the following : 2×10

(a) For which values of x is the matrix $\begin{pmatrix} x & 2 \\ 1 & x-1 \end{pmatrix}$ singular ? Justify your answer.

(b) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find the value of A^3.

P.T.O.

(c) If $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, find the eigenvalues of A^{-1}.

(d) Is the matrix $A = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2}i \\ \sqrt{3}\frac{i}{2} & \frac{1}{2} \end{pmatrix}$ unitary ?

Justify your answer.

(e) Find a unit vector perpendicular to $(-2\hat{i}+3\hat{j}-\hat{k})$ and $(4\hat{i}-2\hat{j})$.

(f) Find the directional derivative of the function $\phi(x, y, z) = x^2 + y^2 - 2z^2$ at the point $(2, -2, 2)$ in the direction of the vector $(2\hat{i}-4\hat{j}-4\hat{k})$.

(g) If $\phi(x, y, z) = x^2 + y^2 - z^2$, find the value of curl (grad ϕ) at any point (x, y, z).

(h) State Gauss divergence theorem.

(i) Consider the Fourier series of the function

$f(x) = |x| \qquad , -\pi \le x \le \pi$

find the value of a_0.

(j) The function

$$f(x) = \begin{cases} k & , -\pi/2 < x < \pi/2 \\ 0 & , \pi/2 < x < 3\pi/2 \end{cases}$$

has the Fourier expansion

$$f(x) = \frac{k}{2} + \frac{2k}{\pi}\left[\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \ldots\right]$$

Show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

2. (a) Solve the following system of equations by Gauss elimination method. 5

$$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$

(b) Determine the inverse of the following matrix by using Gauss-Jordan elimination method or otherwise. 5

$$A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

3. (a) Convert the following matrix into triangular form and hence determine its determinant. 5

$$A = \begin{bmatrix} 3 & 1 & -1 & 0 \\ 2 & 2 & 2 & 1 \\ -1 & 3 & 0 & 4 \\ 8 & 6 & -2 & 2 \end{bmatrix}$$

(b) Use Cramer's rule to solve the following system of equations. 5

$$x_1 + x_2 + x_3 = 4$$

$$x_1 - x_2 + x_3 = 2$$

$$x_1 - 2x_2 \quad = 0$$

4. (a) Reduce the following matrix into diagonal form. 5

$$A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

(b) Find the inverse transformation of the following : 5

$$y_1 = x_1$$

$$y_2 = x_2 \cos\theta + x_3 \sin\theta$$

$$y_3 = -x_2 \sin\theta + x_3 \cos\theta$$

5. (a) (i) Find the volume of the tetrahedron whose Vertices are (1, 3, 6), (3, 7, 12), (8, 8, 9), (2, 2, 8). 5

(ii) Verify whether the following set of vectors are linearly independent.

(4, 2, 9), (3, 2, 1), (−4, 6, 9)

(b) Find $\operatorname{div}\vec{F}$ and $\operatorname{Curl}\vec{F}$ at the point P(1, 2, 3)

If $\vec{F} = x^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$. 5

6. (a) Evaluate the line integral $\int_C F(\vec{r}).d\vec{r}$ 5

where $\vec{F} = [2z, x, -y]$ and C is given by $\vec{r} = [\cos t, \sin t, 2t]$ from $(0,0,0)$ to $(1,0,4\pi)$

(b) Evaluate the double integral 5

$$\int_{1}^{5}\int_{y=0}^{x^2}(1+2x)e^{x+y}\,dx\,dy.$$

7. (a) Use Gauss divergence theorem to evaluate the surface integral $\iint_S \vec{F}.\hat{n}\,dS$ 5

where $\vec{F} = [e^x, e^y, e^z]$ and S is the surface of the cube $|x| \le 1, |y| \le 1, |z| \le 1$.

(b) Evaluate the line integral $\int \vec{F}(\vec{r}).d\vec{r}$ using Stoke's theorem where $\vec{F} = y\hat{i} + \frac{1}{2}z\hat{j} + \frac{3}{2}y\hat{k}$ and C is the circle : $x^2+y^2+z^2=6z,\ z=x+3$. 5

8. (a) Find the Fourier series expansion of : 5

$f(x) = 3x \qquad -\pi < x < \pi$

(b) Find the half-range sine series of the function $f(x)$ defined over the range $0 < x < 2$ as follows : 5

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \end{cases}$$