Total number of printed pages – 6

B. Tech

BSCM 2201

THIRD SEMESTER EXAMINATION – 2005

MATHEMATICS – III

Full Marks : 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** questions from the rest.*

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

(a) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

by separation of variables.

P.T.O.

(b) Prove that $\nabla^2 u = 0$ is invariant under the rotation

$$x^* = x\cos\alpha - y\sin\alpha,$$

$$y^* = x\sin\alpha + y\cos\alpha.$$

(c) Classify the following partial differential equations as elliptic, parabolic or hyperbolic.

(i) $u_{xx} + u_{yy} = 0$

(ii) $4u_{xx} - u_{yy} = 0.$

(d) Write the polar form of the 2-dimensional Laplace equation.

(e) Solve the partial differential equation

$$u_{xy} = u_x.$$

(f) Show that the function $f(z) = \bar{z}$ is not analytic anywhere.

(g) Find the modulus and argument of the complex number

$$z = \frac{1+i}{1-i}.$$

(h) Find the fixed points of the linear fractional transformation

$$w = \frac{z-2}{z+1}.$$

(i) Find the value of the following integral :

$$\int_C \frac{z^2 + 2z + 2}{2z-1}\, dz.$$

where C is the unit circle around the origin.

(j) Find the residues at the poles of the function

$$f(z) = \frac{1}{(z-1)^2\,(z-2)}.$$

2. Find u (x, t) for a string of length π, $c^2 = 1$, initial velocity 0 with initial deflection

$$K\left(\sin x - \frac{1}{2}\sin 2x\right).$$ 10

3. The faces of a thin square plate of side 24 are perfectly insulated. The upper side is kept at 20 °C and the lower side at 0 °C. Find the steady state temperature u (x, y) in the plate. 10

4. Find the deflection u (x, y, t) of a square membrane, with a = b = 1, c = 1, if the initial velocity is zero and the initial deflection u (x, y, 0) is given by

$u(x, y, 0) = f(x, y) = 0.1 \sin 3\pi x \sin 4\pi y.$

$$\left[\text{Use } u(x,y,t)=\sum_m\sum_n\left(B_{mn}\cos\lambda_{mn}t+B^{*}_{mn}\sin\lambda_{mn}t\right)\times \sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\right]$$

10

5. (a) Find an analytic function whose real part is given by $u(x, y) = \sin x \cosh y$. 5

(b) Show that the linear fractional transformation

$$w=\frac{z-i}{z+i}.$$

maps the upper half of the z-plane to the unit circle of the w-plane. 5

6. (a) Find the value of the integral

$$\int_C \frac{z^2-5z+8}{(z-2)^2}\,dz$$

where $C = \{z : |z| = 3\}$. 3

(b) Find the value of the integral

$$\int_C \frac{z^2+5z+6}{z^2-3z+2}\,dz$$

where $C = \{z : |z| = \frac{3}{2}\}$. 3

(c) Find the Laurent's series for the function

$$f(z) = \frac{1}{(z+1)(z-3)}$$

valid in the region $1 < |z| < 3$. 4

7. (a) State the residue theorem. 2

(b) Find the poles of

$f(z) = \operatorname{cosec} z$

and the residues at those poles. 4

(c) Hence find the value of

$$\int_C \operatorname{cosec} z \, dz$$

where $C = \{ z : |z| = 100 \}$ 4

8. Evaluate the following integrals using the residue theorem : 5×2

(a) $$\int_0^{2\pi} \frac{1}{13 + 12\cos\theta} \, d\theta$$

(b) $$\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}.$$