

9T-4456

Total number of printed pages : 7

B. TECH.
BSCM 2202

Fourth Semester Examination, April – 2005

MATHEMATICS-IV

Full Marks : 70

Time : 3 Hours

Answer question No. 1 which is compulsory and any
five questions from the rest.

The figures in the right-hand margin indicate marks
for the questions.

1. Answer the following questions : 2×10
 - (a) Write 0.000623521 in floating point form rounded to 5s.
 - (b) Design a Newton iteration for finding the square root of 3 correct to 2s. Take $x_0 = 1$.

P.T.O.

- (c) Define Δf_0 , ∇f_1 and $\delta f_{1/2}$, where Δ , ∇ and δ are Newton's forward, backward and central difference respectively. Are they equal ?
- (d) Write down the Simpson's $1/3^{\text{rd}}$ rule for numerical integration.
- (e) For a LPP, define the following terms :
slack, basic variables.
- (f) A bag has 20 tickets numbered 1, 2, 3 ..., 20. If you pick a ticket at random, what is the probability that the ticket has a prime number printed on it.
- (g) A die is cast twice and let X, Y be the numbers obtained, find the probability of the event that $X < Y$.
- (h) A fair coin is tossed 100 times. If X is the number of heads obtained (which is a random variable) what is the expectation value of X.

- (i) A book has 4 misprints per page (on average). What is the probability that a page opened at random will have no misprint on it.
- (j) Define a confidence interval for an unknown parameter θ .

Answer the following questions : $10 \times 5 = 50$

2. (a) Use the method of iteration to find a real root of $\sin^2 x = x^2 - 1$ correct upto four significant figures. 5
- (b) Given the table of values :

x	150	152	154	156
$y = \sqrt{x}$	12.247	12.329	12.410	12.490

Find the value of $\sqrt{155}$ using Lagrange's interpolation formula. 5

3. (a) Evaluate the following integral numerically using Simpson's $1/3^{\text{rd}}$ rule : 5

$$I = \int_0^{0.4} x e^{-x^2} dx \quad (h=0.05)$$

Compare your result with the exact value of the integral.

(b) Use the formula :

$$f'(x_0) = \frac{1}{h} \left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \dots \right] f_0$$

to evaluate $f'(x)$ at $x = 1.4$ where f is given by the table : 5

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

4. (a) Use Gauss-Siedel method to solve the system of equations : 5

$$x_1 + 9x_2 - 2x_3 = 36$$

$$2x_1 - x_2 + 8x_3 = 121$$

$$6x_1 + x_2 + x_3 = 107$$

(b) Use the Runge-Kutta method to solve the following : 5

$$10y' = x^2 + y^2, \quad y(0) = 1$$

for the interval $0 < x \leq 0.4$, $h = 0.1$

5. (a) Describe and graph the region in the first quadrant of the xy -plane determined by the following inequalities : 5

$$2x + y \geq 10$$

$$y \geq 4$$

$$10x + 15y \leq 150$$

and maximize $f(x) = 30x + 10y$ in this region.

(b) If a pair of dice are cast, find the probability that : 5

(i) Sum of points is greater than 4

(ii) Sum of points is greater than 10.

6. (a) The probability of a student passing a test is $2/3$. If the student appears in 4 tests, find the probability of : 5

(i) His passing at least 3 tests

(ii) His passing 3 consecutive tests.

(b) A probability density function is defined as follows : 5

$$f(x) = \begin{cases} k(1-x^2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k .

7. (a) A coin is tossed 400 times. Find the probability of getting more than 216 heads. You may use the following normal table : 5

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-1/2t^2} dt$$

x	1.0	1.3	1.5	1.65	2.33
$\Phi(x)$	0.84	0.90	0.93	0.95	0.99

- (b) Find the correlation coefficient for a sample of size 8 where the values of the random variables X and Y are given below : 5

X	2	4	6	8	10	12	14	16
Y	2	2	4	4	6	6	8	8

8. (a) Find the regression line of Y on X from the following table : 5

X	5	6	7	8	9
Y	7	10	12	9	12

- (b) Determine a 99% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 25$ using a sample size of 400 and mean 10. [use $c = 2.576$] 5