DISCRETE MATHEMATICAL STRUCTURE

Full Marks – 70

Time : 3 Hours

The figures in the right hand margin indicate full marks for the questions.

Answer Q. No. 1 which is compulsory and any ***five*** *questions from the remaining.*

1. Answer the following questions : 2×10

(a) Translate each of the following statement into logical expressions using predicate, quantifiers and logical connectives.

(i) All students in your class can solve quadratic equations.

P.T.O.

(ii) Some students in your class does not want to be rich.

(b) Give the recurssive definition of the sequence $\{a_n\}$ where $a_n = 3^n$.

(c) Find the number of bit strings of length n containing exactly r number of 1's.

(d) Construct the Hasse diagram of the partial order relation divides in the set of all divisors of 24.

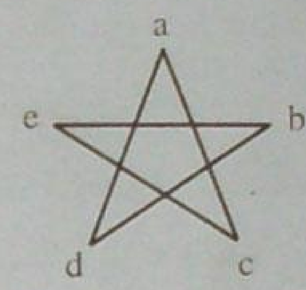
(e) Find the transitive closure of the relation $\{(1,2), (2,1), (2,3), (3,4), (4,1)\}$ defined on the set $\{1,2,3,4\}$.

(f) Define degree of a vertex and diameter of a graph.

(g) Do you agree $(ab)^{-1} = a^{-1} b^{-1}$ for a group which contain a and b ? Justify.

(h) How many edges are there in a graph with 10 vertices each of degree 6.

(i) Is the following graph Eulerian ? Justify.



(j) Find the complement of each element if exist of the lattice D_{24} (set of all divisors of 24) with the partial order relation "divides".

2. (a) Show that the following propositions are logically equivalent. 5

$(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

(b) Show that $1^3 + 2^3 + \ldots\ldots + n^3 = [\, n\,(n+1)\,/2]^2$ by method of induction when n is a positive integer. 5

3. (a) Show that among any n +1 positive integers not exceeding 2n there must be an integer that divides one of the other integers.

(b) Find the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 17$, where each $x_i \geq 0$, and $x_1 \leq 3$, $x_2 \leq 4$, $x_3 \leq 5$ and $x_4 \leq 8$. 5

4. (a) Prove that for R to be one equivalence relation on a set A, the following statements are equivalent : 5

i) a R b, ii) [a] = [b], iii) [a] $\cap$ [b] = ϕ.

(b) For

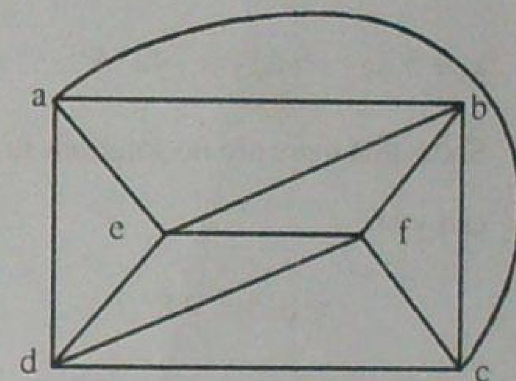
$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

find the following matrices : 5

(i) R^{-1}, (ii) $\overline{R}$, (iii) R^2

5. (a) Prove that a connected multigraph has an Eulerian circuit if and only if each of its vertices has even degree. 5

(b) Find the chromatic number of the graph given below : 5



6. (a) Prove that if (A, *) is a group, B a subset of A with finite number of elements and * is a closed operations on B then (B, *) is a subgroup. 5

(b) For Z_n = { 0, 1, 2, ..., n −1} with $\odot$ a binary operation on Z_n such that $a \odot b$ = the remainder of ab divided by n.

then find

i) the table for operation $\odot$ when no 4.

ii) show that $(Z_n, \odot)$ is a semigroup for any n. 5

7. (a) Find all solutions of the recurrence relation

$$a_n + 5\,a_{n-1} - 6\,a_{n-2} = 42\,(4^n).$$ 5

(b) Show that there are no solutions in integers x and y of 5

$$x^2 + 3y^2 = 8.$$

8. (a) Prove that both meet and join operations defined on a lattice are associative. 5

(b) Show that a tree with n vertices has n–1 edges. 5
