FIFTH SEMESTER EXAMINATION – 2005

OPTIMISATION IN ENGINEERING

Full Marks – 70

Time : 3 Hours

The figures in the right hand margin indicate full marks for the questions.

*Answer question No. **1** which is compulsory and any **five** from the rest.*

1. Answer the following questions : 2×10

   (i) Define a spanning tree in network model.

   (ii) What is a saddle point in a pay off matrix in game theory ?

   (iii) Find the initial basic feasible solution of the following transportation problem by north-west corner method :

P.T.O.

Ware house → Factory ↓	W_1	W_2	W_3	W_4	Factory capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Ware house requirement	5	8	7	14	34

(iv) Determine the feasible space for the following constraints, given that $x_1, x_2 \geq 0$

$$-3x_1 + x_2 \leq 6$$

$$x_1 - 2x_2 \geq 5$$

$$x_1 - x_2 \leq 0$$

(v) Write the dual of the following L P P

Maximize $\mathbb{Z} = x_1 + 5x_2 + 3x_3$

subject to : $x_1 + 2x_2 + x_3 \leq 3$

$2x_1 - x_2 \leq 4$

$x_1, x_2, x_3 \geq 0$

(vi) Define the following terms in the theory of sequencing :

(a) processing time.

(b) total elapsed time.

(vii) Define degeneracy in L P P

(viii) Determine the direction of increase in $\mathbb{Z}$ in the following cases

(a) Maximize $\mathbb{Z} = x_1 - x_2$

(b) Maximize $\mathbb{Z} = -3x_1 + x_2$

(ix) What is the condition that the solution of a L P P is unbounded in simplex method.

(x) Solve the following assignment problem by Hungarian method.

Jobs

Persons	J_1	J_2	J_3
P_1	15	10	9
P_2	9	15	10
P_3	10	12	8

2. (a) Solve the following LPP by graphical method. 5

Maximize $Z = 5x_1 + 4x_2$

Subject to : $6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

$-x_1 + x_2 \le 1$

$x_2 \le 2$

$x_1, x_2 \ge 0$

(b) Solve the following LPP by two phase method. 5

Maximize $Z = 2x_1 + 2x_2 + 4x_3$

$3x_1 + 4x_2 + 2x_3 \ge 8$

$2x_1 + x_2 + x_3 \le 2$

$x_1, x_2, x_3 \ge 0$

3. Find the optimum solution to the LPP.

Maximize $Z = 15x_1 + 45x_2$

Subject to : $x_1 + 16x_2 \le 240$

$5x_1 + 2x_2 \le 162$

$x_2 \le 50$

$x_1, x_2 \ge 0$

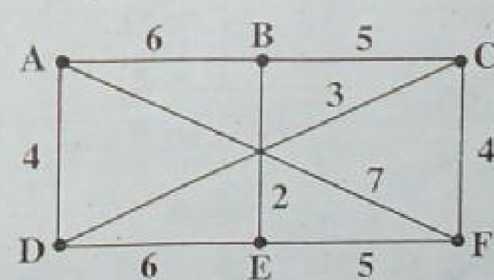
In the optimum solution $Z = c_1x_1 + c_2x_2$ if c_2 is kept fixed at 45, determine how much c_1 can be changed without affecting the above solution. 10

4. A company has four warehouses and 6 stores ; the cost of shipping of one unit from warehouse i to store j is C_{ij}. If –

$$C = (C_{ij}) = \begin{pmatrix} 7 & 10 & 7 & 4 & 7 & 8 \\ 5 & 1 & 5 & 5 & 3 & 3 \\ 4 & 3 & 7 & 9 & 1 & 9 \\ 4 & 6 & 9 & 0 & 0 & 8 \end{pmatrix}$$

and the requirements of the six stores are 4, 4, 6, 2, 4, 2 units and the quantities at the warehouses are 5, 6, 2, 9, find the minimum cost of transportation. 10

5. (a) Write an algorithm to find the minimum spanning tree from a weighted network. Hence find out the minimum spanning tree of the following weighted network. 5



Ware house → / Factory ↓	W_1	W_2	W_3	W_4	Factory capacity
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(iv) Determine the feasible space for the following constraints, given that $x_1, x_2 \geq 0$

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(v) Write the dual of the following L P P

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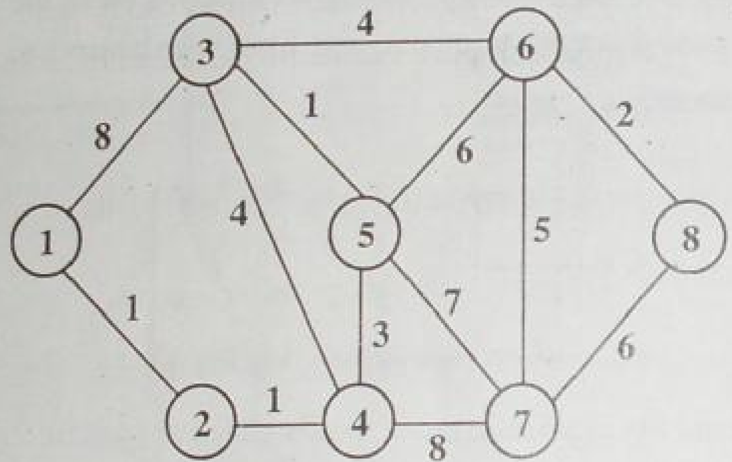
(x) Solve the following assignment problem by Hungarian method.

Persons \ Jobs	J_1	J_2	J_3
P_1	15	10	9
P_2	9	15	10
P_3	10	12	8

(b) Write the Dijkstra's algorithm of shortest route. Hence determine the shortest route between the node (1) to (8). 5



6. Describe the branch and bound algorithm in Integer Programming Problem. Hence find the optimum solution to the following IPP.

Maximize $Z = x_1 + x_2$

Subject to : $3x_1 + 2x_2 \leq 12,$

$x_2 \leq 2,$

$x_1 \geq 0, x_2 \geq 0$ and

are integers. 10

7. (a) Solve the following 2×4 game graphically :

		I	II	III	IV
A	I	2	2	3	–1
	II	4	3	2	6

5

(b) Arrival at a telephone booth are considered to be poisson's distribution with an avarage time of 10 minutes between two consecutive arrivals. If the length of phone calls assumed to distributed exponentially with mean 3 minutes then :

(i) What is the probability that a person arriving at the booth will have to wait ?

(ii) What is the avarage length of the queues that is formed time to time ? 5

8. Write short notes on any *two* : 5×2

(i) Monte-Carlo simulation

(ii) Knapsack problem

(iii) Sequencing problem.