First Semester Examination – 2007

MATHEMATICS – I

Full Marks – 70

Time : 3 Hours

Answer Question No. **1** which is compulsory and
any **five** from the rest.

The figures in the right hand margin indicate
full marks for the questions.

1. Answer the following questions precisely :

2×10

(a) Write the parallel asymptote of the curve

$$y = \frac{x^2}{x^2 + 1}.$$

P.T.O.

(b) Write the general solution of the differen-tial equation $y' + y = e^{-x}$.

(c) Write the particular solution of the differential equation $y'' + y' = 2$ in the most general method by undetermined coefficient method.

(d) If the equation $y'' + P(x)y' + Q(x) = 0$ has series solution about the ordinary point $x = a$, then write the conditions that $P(x)$ and $Q(x)$ have to satisfy.

(e) If the equation $y'' + P(x)y' + Q(x) = 0$ has series solution about the regular singular point $x = a$, then write the conditions that $P(x)$ and $Q(x)$ have to satisfy.

(f) Write the solution of the differential equation $y' + y = 0$ in series.

(g) What is the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} \dfrac{(x-2)^n}{n!}$.

(h) What is the value of $P_{2n+1}(0)$, the Legendree polynomial of degree $2n+1$.

(i) Write the polynomial expression of $P_2(x)$, the Legendree polynomial of degree 2.

(j) Find the Laplace transform $L(\sin(wt))$ using the result $L(e^{iwt}) = \dfrac{1}{s - iw}$.

2. Solve the following problems :

(a) Find the radius curvature of the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the point $(a, 0)$.   5

(b) Find the asymptote to the curve $x^3 - y^3 = 3ax^2$.   5

3. Answer the following questions as per the instruction :

(a) Solve the Bernoulli's equation

$$y' - 2xy = 2xy^2.$$ 5

(b) A tank of 100 gallons capacity is initially full of water. Pure water is allowed to run into the tank at the rate of 1 gallon per minute, and at the same time brine containing 0.25 pounds of salt per gallon flows into the tank at the rate of 1 gallon per minute. If the mixture is allowed to flow out at the rate of 2 gallons per minute after perfect mixing, then find the amount of salt in the tank after t minutes.

5

4. Solve the following initial value problems :

(a) $y'' + 4y = 4\cos(2x)$ with $y(0) = 0$ and $y'(0) = 2$ using method of undetermined coefficient. 5

(b) $y'' - 5y' + 6y = e^{4x}$ with $y(0) = \frac{1}{2}$ and $y'(0) = 2$ using method of variation of parameter.

5

5. Answer the following questions according to the instruction :

(a) Solve the equation $(x-1)y'' - xy' + y = 0$ by reducing the order using $y = e^x$ as one of the solution. 5

(b) Solve Cauchy-Euler equation

$$x^2y'' - 5xy' + 8y = 0$$

by reducing into constant coefficient differential equation. 5

6. Answer according to the instruction :

   (a) Find the series solution of the differential equation $y'' - 9y = 0$ with $y(0) = 1$ and $y'(0) = 0$.   5

   (b) Prove the identity   5

   $$J_{-\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \cos(x).$$

7. Answer according to the instruction :

   (a) Find the Laplace transform of the function

   $$f(t) = \begin{cases} \left(\dfrac{\alpha}{a}\right)t, & 0 < t < a \\ \left(\dfrac{\alpha}{a}\right)(2a - t), & a < t < 2a \\ 0, & \text{otherwise} \end{cases}$$

   where α and a are constants.   5

   (b) Find the inverse Laplace transform of

   $$F(s) = \frac{9}{s^2(s^2 - 9)}.$$ 5

8. Answer the following questions according to the instruction :

   (a) Solve the initial value problem $y'' + y = 2$ with $y(0) = 0$ and $y'(0) = 2$ using Laplace transform.   5

   (b) If $f * g = \int_0^t f(t - \gamma)g(\gamma)\,d\gamma$, then show that

   $$f * g = g * f.$$ 5