Total number of printed pages – 7

B. Tech/B. Arch

BSCM 2102/SCM 2002

Second Semester Examination – 2007

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

(a) Give the examples of two matrices A and B such that $A \neq 0$, $B \neq 0$ but $AB = 0$.

(b) Are the vectors (1, 0, 1), (0, 1, 1) and (1, 1, 1) linearly independent ?

P.T.O.

(c) For what value of c the equations $x+y=7$ and $2x-cy=14$ have infinitely many solutions ?

(d) If A is a Hermitian matrix, prove that its diagonal elements are real.

(e) Find the component of $\boldsymbol{a} = [4, 0, -3]$ in the direction of $\boldsymbol{b} = [1, 1, 1]$.

(f) If $\boldsymbol{a} = [4, 0, -3]$, $\boldsymbol{b} = [1, 1, 1]$ and $\boldsymbol{c} = [1, 2, 1]$ find $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$.

(g) Prove that curl(grad f) = 0.

(h) If $f(x, y, z) = e^x \cos y + e^{-y} \sin z + \cos z$, find $\Delta^2 f$.

(i) Find by inspection the Fourier series of $1 + \sin^2 2x + \cos^2 3x$ in the range $-\pi < x < \pi$.

(j) If $m \neq n$ then prove that

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = 0.$$

2. (a) If $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ and $B^T = \begin{bmatrix} 2 & 5 & 1 & 8 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 3 & 6 \end{bmatrix}$,

verify that $(AB)^T = B^T A^T$. 5

(b) Find the rank of the matrix 5

$$A = \begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

3. (a) Solve the following systems of equations by Cramer's rule : 5

$$x+2y+3z=20$$

$$7x+3y+z=13$$

$$x+6y+2z=0.$$

(b) Find the inverse of the matrix

$$A=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

by Gauss-Jordan method. 5

4 (a) Find the eigen values and eigen vectors of the matrix 5

$$A=\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Find a matrix A such that

$$x^T Ax = (x_1 - x_2 + 4x_3)^2 - 4(x_2 - x_4)^2.$$ 5

5. (a) If $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two vectors prove that $|a+b|^2 + |a-b|^2 = 2(|a|^2 + |b|^2)$. 5

(b) Find the volume of the tetrahedron if its vertices are (1, 3, 6), (3, 7, 12), (8, 8, 9) and (2, 2, 8). 5

6. (a) Find the directional derivative of 5

$$f(x,y,z,) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

at the point (3, 0, 4) in the direction of $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$.

(b) Evaluate the line integral

$$\int_C F(r) \, . \, dr$$

if $F = [xy, x^2 y^2]$ and C, the quarter circle from (2, 0) to (0, 2) with centre at (0, 0). 5

7. (a) Evaluate the double integral 5

$$\int_0^{\pi/4} \int_0^{\cos y} x^2 \sin y \, dx \, dy.$$

(b) Using Gauss divergence theorem, evaluate the integral 5

$$\iint_S F \, . \, n \, dA$$

if $F = [x^3, y^3, z^3]$ and S is the sphere $x^2 + y^2 + z^2 = 9$.

8. (a) Find the Fourier series of 5

$f(x) = x^2 + 2 \sin 2x$

in the range $-\pi < x < \pi$.

(b) Find a Fourier cosine series for

$f(x) = x$ valid in $0 < x < 1$. 5