

Total number of printed pages – 7 B. Tech/B. Arch  
BSCM 2102/SCM 2002

Second Semester Examination – 2007

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10
- (a) Give the examples of two matrices A and B such that  $A \neq 0$ ,  $B \neq 0$  but  $AB = 0$ .
- (b) Are the vectors  $(1, 0, 1)$ ,  $(0, 1, 1)$  and  $(1, 1, 1)$  linearly independent ?

P.T.O.

(c) For what value of  $c$  the equations  $x+y=7$  and  $2x-cy=14$  have infinitely many solutions?

(d) If  $A$  is a Hermitian matrix, prove that its diagonal elements are real.

(e) Find the component of  $\mathbf{a} = [4, 0, -3]$  in the direction of  $\mathbf{b} = [1, 1, 1]$ .

(f) If  $\mathbf{a} = [4, 0, -3]$ ,  $\mathbf{b} = [1, 1, 1]$  and  $\mathbf{c} = [1, 2, 1]$  find  $[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]$ .

(g) Prove that  $\text{curl}(\text{grad } f) = 0$ .

(h) If  $f(x, y, z) = e^x \cos y + e^{-y} \sin z + \cos z$ , find  $\Delta^2 f$ .

(i) Find by inspection the Fourier series of  $1 + \sin^2 2x + \cos^2 3x$  in the range  $-\pi < x < \pi$ .

(j) If  $m \neq n$  then prove that

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = 0.$$

2. (a) If  $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$  and  $B^T = \begin{bmatrix} 2 & 5 & 1 & 8 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 3 & 6 \end{bmatrix}$ ,

verify that  $(AB)^T = B^T A^T$ . 5

(b) Find the rank of the matrix 5

$$A = \begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

3. (a) Solve the following systems of equations by Cramer's rule : 5

$$x+2y+3z=20$$

$$7x+3y+z=13$$

$$x+6y+2z=0.$$

- (b) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

by Gauss-Jordan method. 5

- 4 (a) Find the eigen values and eigen vectors of the matrix 5

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}.$$

- (b) Find a matrix A such that

$$x^T A x = (x_1 - x_2 + 4x_3)^2 - 4(x_2 - x_4)^2. \quad 5$$

5. (a) If  $a$  and  $b$  are any two vectors prove that  $|a+b|^2 + |a-b|^2 = 2(|a|^2 + |b|^2)$ . 5

- (b) Find the volume of the tetrahedron if its vertices are  $(1, 3, 6)$ ,  $(3, 7, 12)$ ,  $(8, 8, 9)$  and  $(2, 2, 8)$ . 5

6. (a) Find the directional derivative of 5

$$f(x,y,z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

at the point  $(3, 0, 4)$  in the direction of  $a = i + j + k$ .

(b) Evaluate the line integral

$$\int_C F(r) \cdot dr$$

if  $F = [xy, x^2y^2]$  and  $C$ , the quarter circle from  $(2, 0)$  to  $(0, 2)$  with centre at  $(0, 0)$ .

5

7. (a) Evaluate the double integral 5

$$\int_0^{\pi/4} \int_0^{\cos y} x^2 \sin y \, dx \, dy.$$

(b) Using Gauss divergence theorem, evaluate the integral 5

$$\iiint_S F \cdot n \, dA$$

if  $F = [x^3, y^3, z^3]$  and  $S$  is the sphere  $x^2 + y^2 + z^2 = 9$ .

8. (a) Find the Fourier series of 5

$$f(x) = x^2 + 2 \sin 2x$$

in the range  $-\pi < x < \pi$ .

(b) Find a Fourier cosine series for

$$f(x) = x \text{ valid in } 0 < x < 1. \quad 5$$

\_\_\_\_\_

IWL