

Third Semester Examination – 2007

MATHEMATICS – III

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory  
and any five from the rest.*

*The figures in the right-hand margin  
indicate marks.*

1. Answer the following questions precisely :

2×10

(a) Solve the partial differential equation

$$u_{xy} - u_y = 0.$$

P.T.O.

(b) Solve the partial differential equation

$$u_{xx} - u_{yy} = 0 \text{ by separation of variables.}$$

(c) Write the D'Alemberts solution of the

$$\text{wave equation } u_{tt} = c^2 u_{xx} \text{ with } u(x, 0) = f(x)$$

$$\text{and } u_t(x, 0) = g(x).$$

(d) Classify the partial differential equation

$$u_{xx} + 4u_{xy} - u_{yy} = 0 \text{ as elliptic, parabolic}$$

and hyperbolic.

(e) Write the transform in general form which

can transfer the partial differential equa-

$$\text{tion } u_{xx} + 4u_{xy} + 4u_{yy} = 0 \text{ into canonical}$$

form.

(f) Is the function

$$f(z) = \begin{cases} \frac{xy}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

analytic at  $z = 0$  and why ?

(g) How many conditions are required to deter-

mine all the unknown present in a linear

$$\text{fractional transformation } T(z) = \frac{az + b}{cz + d}$$

where  $ad - bc \neq 0$  and why ?

(h) How many fixed points does a linear

fractional transformation have and why ?

(i) Is there any analytic function  $f(z)$  whose

imaginary part is  $v(x, y) = x^3 + 2y$  and

why ?

(j) Find the poles of the function  $f(z) = \sec(z)$ .

2. A elastic string of length  $l$  has its ends at  $x = 0$  and  $x = l$  fixed. If the point  $x = \frac{l}{3}$  is drawn aside a small distance  $h$  and released at time  $t = 0$ , then find the displacement  $u(x, t)$  of the string at any subsequent time  $t > 0$ . 10

3. An insulated rod of length  $l$  has its ends A and B maintained at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  respectively until steady state conditions prevail. If the end B is suddenly reduced to  $0^\circ\text{C}$  and maintained at  $0^\circ\text{C}$ , then find the temperature distribution  $u(x, t)$  at a distance  $x$  from the end A at any subsequent time  $t > 0$ . 10

4. Find the steady state temperature distribution  $u(x, y)$  in the uniform unit square  $0 \leq x \leq 1$  and

$0 \leq y \leq 1$  when edge  $y = 1$  is maintained at the temperature  $x(1 - x)$  and the other three edges being thermally insulated. 10

5. Write the answer according to the instruction :

(a) Find the harmonic conjugate of  $u(x, y) = \cos(x)\sinh(y)$ . 5

(b) Find the linear fractional transform which maps lower half plane into the unit disk. 5

6. Evaluate the following integrations over the prescribed simple closed path  $C$  :

(a)  $\int_C \frac{z}{(z-2)^2} dz$  where  $z = \{z : |z| = 3\}$ . 3

(b)  $\int_C \frac{f(z)}{(z-2)^2} dz$  where  $z = \{z : |z| = 1\}$

and  $f(z)$  is analytic over boundary and interior of the simple closed path  $C$ . 2

(c)  $\int_C \frac{2z-3}{(z-2)(z-1)} dz$  where  $C = \{z : |z| = 3\}$ . 5

7. Evaluate as per the instruction :

(a) Find the Laurent series of the function

$f(z) = \frac{z}{(z+1)(z+2)}$  which is valid in the

region  $\frac{1}{2} < |z+1| < \frac{3}{4}$ . 5

(b) Find the residue of the function

$f(z) = \frac{z+1}{(z-1)^2(z+2)}$  at  $z = 1$ . 5

8. Evaluate the following integrations using residue theorem :

(a)  $\int_0^{\infty} \frac{dx}{1+x^2}$  5

(b)  $\int_0^{2\pi} \frac{d\theta}{2+\cos(\theta)}$  5

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