Fifth Semester Examination – 2007

AUTOMATA THEORY

Full Marks – 70

Time – 3 Hours

Answer Question No. **1** *which is compulsory*

*and any* **five** *from the rest.*

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The figures in the right-hand margin

indicate marks.

1.   Answer the following questions :   2 × 10

   (a)   Given an alphabet Σ, what do you mean by
        a language L over Σ ?

   (b)   Devise a DFA which accepts all strings
        ending with 1. (Assume that $\Sigma = \{0, 1\}$)

   (c)   Define a NFA.

(d) You have a DFA M_1 which accepts a language L_1 and another DFA M_2 which accepts the language L_2. Devise a $\in$-NFA which accepts the language $L_1 \cup L_2$.

(e) Find the $\in$-closure of the state $\{q_0\}$ from the following diagram :
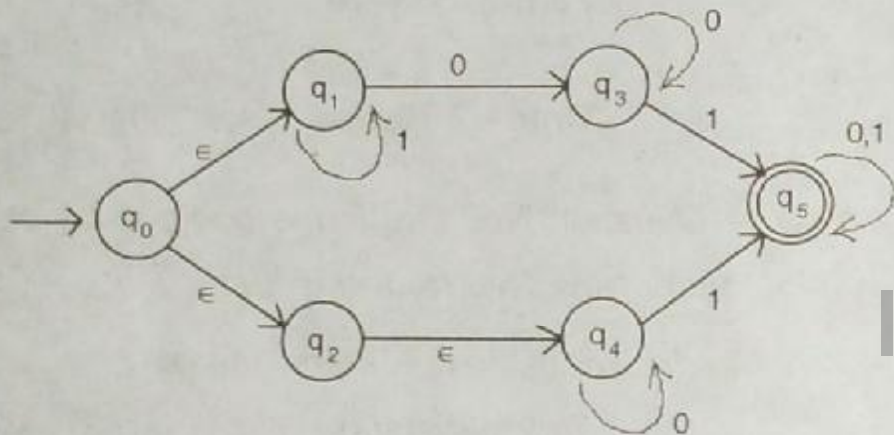
(f) Define a pushdown automaton.

(g) Find the language generated by the following context-free grammar :

$$S \to aSa \mid bSb \mid a \mid b$$

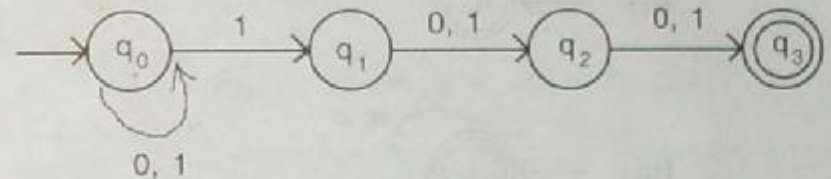(h) When is a context-free grammar said to be ambiguous ?

BCSE 3308   2   Contd.

(i) When is a language L said to be Turing recognizable ? Turing decidable ?

(j) Let f, g, h, k : $\mathbf{N} \to \mathbf{N}$. Show that if $f = O(h)$ and $g = O(k)$, then $fg = O(hk)$.

2. (a) Devise a DFA which accepts all binary strings which are divisible by 4.   5

(b) Convert the following NFA to its equivalent DFA.   5

3. (a) From the diagram of a DFA given below, devise a regular grammar which generates the language of the DFA.   4
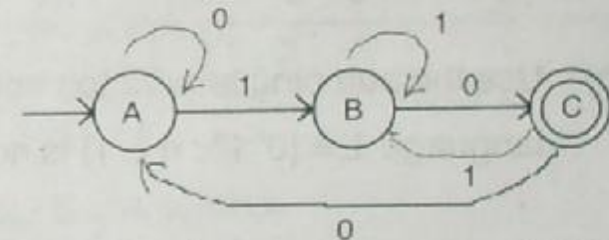
(b) Describe the regular expression corresponding to the language generated by the DFA in 3(a) above.   3

BCSE 3308   3   P.T.O.

(c) Find the languages generate by the
following DFA's : 3

(i)

(ii) → q_0

(iii)

4. (a) State and prove the pumping lemma for a
regular language. 5

(b) Use the pumping lemma to show that the
language $L = \{0^n 1^n : n \geq 1\}$ is not regular.

5

5. (a) State and prove the pumping lemma for
context-free languages. 5

(b) Show that the language $L = \{a^n c b^n : n \geq 1\}$
is accepted by a PDA. 5

6. (a) Define a Turing machine. 5

(b) Devise a Turing machine which on a given
pair of positive integers (m, n) as input, will
produce m + n as output. [Assume that the
positive integers are represented by a
unary alphabet {1}]. 5

Therefore the input on the tape is :

m 1's   n 1's

The output should be

m+n 1's

□ = blank symbol.

7. (a) Show that the set of all Turing recognizable
languages is countable. 5

(b) Show that if a language L is decidable so is its complement. 5

8. (a) When is a function $f : \Sigma^* \to \Sigma^*$ said to be computable ? (Where Σ is a given alphabet) 2

(b) When is a language A said to be mapping reducible to another language B (both over the same alphabet) ? 2

(c) Define the calss NP. Describe three problems belonging to the class NP. 6

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