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B. Tech
BSCM 3301

Fifth Semester Examination – 2007

DISCRETE MATHEMATICAL STRUCTURES

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory
and any **five** from the rest.

The figures in the right-hand margin
indicate marks.

1. Answer the following questions : 2 × 10
- (i) Define scope of a quantifier and give an example.
 - (ii) What is Euclidian algorithm to find the GCD of two integers ?

P.T.O.

(iii) What is lexicographic ordering on $N \times N$, where N is the set of natural numbers?

(iv) Give a recursive definition of the set of all positive integers congruent to 2 modulo 3.

(v) How many positive integers less than 1000 exist which are divisible by 7 but not by 11?

(vi) What is symmetric closure of the relation $R = \{ (a, b) \mid a > b \}$?

(vii) What are the principles of drawing Hasse diagram of a partial ordering relation?

(viii) Define chromatic number of a simple graph and that of a planar graph.

(ix) If $(\{a, b\}, *)$ be a semigroup where $a * a = b$ then can you say that $b * b = b$? Justify your answer.

(x) Let a, b are two elements of a lattice (A, \leq) . Show that $a \wedge b = b$ iff $a \vee b = a$.

2. (a) Show that the hypotheses $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the inclusion $p \vee s$. 5

(b) Using mathematical induction show that $n^3 - n$ is divisible by 3 whenever n is a positive integer. 5

3. (a) Using Warshall's algorithm find the transitive closure of the relation whose matrix

representation is given below. . 5

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) Let R be a relation on the set of ordered pairs of positive integers such that

$((a, b), (c, d)) \in R$ if and only if $ad = bc$

Show that R is an equivalence relation. 5

4. (a) Show that the relation 'set inclusion' on the power set of S is a partial ordering relation.

Draw the Hasse diagram of the relation

where $S = \{a, b, c\}$. 5

(b) How do you differentiate between the maximal element and greatest element of a

poset? Show that there is exactly one greatest element of a poset, if such an element exists. What about maximal element?

5

5. (a) Solve the following recursive relation

$$a_r = a_{r-1} - a_{r-2}, \text{ given that } a_1 = 1 \text{ and } a_2 = 0$$

5

(b) Use generating function to find the number of ways to select r objects of n different kinds if we must select at least one object of each kind. 5

6. (a) Define a tree. Show that an undirected graph is a tree if and only if there is a unique simple path between any two of its vertices. 5

- ✓(b) Show that a connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree.

5

- ✓7. ✓(a) Let $(A, *)$ be a monoid such that for every $x \in A$, $x * x = e$, where e is the identity element. Show that $(A, *)$ is an abelian group.

5

- ✓(b) Let (A, \leq) be a distributive lattice. If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a , then show that $x = y$.

5

- ✓8. ✓(a) Explain the principle of inclusion-exclusion. Using this principle, find out the number of solutions for the equation $x_1 + x_2 + x_3 = 13$,

where x_1, x_2 and x_3 are non-negative integers less than six.

5

- ✓(b) Find the total number of strings of 10 ternary digits (0, 1 or 2) which contain exactly two 0s, three 1s and five 2s.

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