Sixth Semester Examination – 2007

DIGITAL SIGNAL PROCESSING

Full Marks – 70

Time : 3 Hours

Answer the questions either from Set – A or Set – B but not from both.

P.T.O.

Set – 'A'

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

(a) What is Nyquist rate ? Compute the Nyquist rate for the given analog signal.

$x_a(t) = 10\cos 50\pi t + 20\sin 200\pi t - 10\cos 100\pi t$

(b) Find the discrete-time-signal having the following Fourier transform :

$$X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le 1.2 \\ 1, & 1.2 < |\omega| \le \pi \end{cases}$$

(c) Determine the even and odd parts of the following real sequence :

$x(n) = \{-2\ 1\ 2\ 5\ \underset{\uparrow}{0}\ 0.1\ 6\ 3\}$

(d) Show that a causal real sequence x(n) can be fully recovered from its even part $x_{ev}(n)$ for all n, whereas it can be recovered from its odd part $x_{odd}(n)$ for all $n > 0$.

(e) What is the unit of z^{-1} ?

(f) What is time reversal property of Z-transform ? Discuss.

(g) What is the link between autocorrelation and power spectral density ?

(h) Draw the structure of the following difference equation :

$y(n) = 4x(n) + 6x(n-1) + 4x(n-2) - 2y(n-1)$

(i) What is linear phase characteristic of FIR filter ? Where is it useful ?

(j) Write two advantages of non parametric method of power spectrum estimation.

2. (a) Test the following discrete systems as per the given direction : 2

(i) $y(n) = \alpha x(-n)$, Linearity test ;

(ii) $y(n) = \beta + \sum_{l=-5}^{l=5} x(n-l)$, Causality test.

(b) Test whether the system $y(n) = x(-n+2)$ is shift invariant and stable. 2

(c) The impulse response of a LTI system is $h(n) = \{2, 1, 4, 2\}$. Find the response of the system if the input is $x(n) = \{1, 2, 3, 4\}$. Use graphical method. 6

3. (a) Obtain the circularly convolved output from the following data : 5

$N = 3, x(n) = -1, 2, 4$ and $h(n) = 2, 1, 2$

(b) Let y(n) be the sequence obtained by a linear convolution of two causal finite duration sequences h (n) and x (n). For the given y (n) and h (n), determine x (n). 5

$y(n) = \{2, 8, 20, 40, 60, 68, 62, 40\}$;

$h(n) = \{2, 4, 6, 8\}$

4. (a) Find the impulse response of the system described by $x(n) = y(n-1) + 2y(n-2)$. Assume zero initial conditions. 4

(b) Define 'Z-transform'. Find the inverse Z-transform for the following function : 4

$X(z) = \frac{1+4z^{-2}}{1+z^{-2}}$. Use long division method.

(c) Write circular convolution property of DFT. 2

5. (a) Distinguish between digital FIR and IIR filters. Discuss window based design method for design of FIR filters. 4

(b) Find the system function H (z) of the digital Butterworth filter that meets the following specifications :

(i) 2-dB ripple in the passband $0 \le |\omega| \le 0.3\pi$

(ii) At least 30 dB attenuation in the stop band $0.3\pi \le |\omega| \le \pi$

Use bilinear transformation method. 6

6. (a) Show how an inverse DFT can be obtained by using direct DFT method. Draw the flow

graph for a 4-point FFT by DFT method. Explain how the same flow graph can be used to compute inverse DFT. 5

(b) Compute the IDFT of the sequence $X(k) = \{4, 2 + j, 2, 2 - j\}$. 5

7. (a) Using impulse invariance method obtain the digital transfer function and the corresponding filter structure

$H_a(s) = \{1/(s + 0.5)(s^2 + 0.5s + 2)\}$. Assume $T = 1$ sec.

(b) Draw the corresponding IIR filter structure. 2

(c) Define 'analog frequency' and 'digital frequency'. 2

8. (a) Why power spectrum estimation is required in signal processing ? List two applications. 5

(b) Explain Blackman and Tukey method of power spectrum estimation. What are its advantages over Bartlett method ? 5

Set – 'B'

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

(i) What are discrete time and continuous time signals ?

(ii) Differentiate between odd and even signals.

(iii) What is an impulse function ?

(iv) What is the Fourier Transform of sin (t) ?

(v) What is a causal system ?

(vi) What do you mean by periodic or circular convolution ?

(vii) What are twiddle factors ?

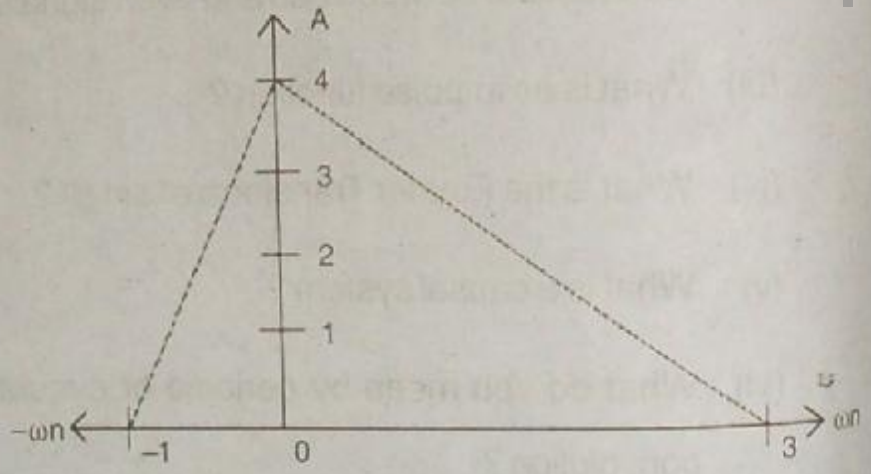
(viii) What are advantages and disadvantages of digital filters over analog filters ?

(ix) What is quality of power spectrum estimation ?

(x) What do you mean by parametric methods for power spectrum estimation ?

2. (a) Find even and odd parts of the function $x(n) = A \sin \omega n$. 5

(b) Plot the even and odd parts of the function



(Fig. 1)

3. Given $y(n) = \delta(n)$, sketch the following signals : 10

(a) $y(n - 6)$

(b) $y(n + 4)$

(c) $y(n^2)$

(d) $y[(n - 6)^2]$

4. Convolve the followings : 5+5

(a) $x(n) = u(n + 1) - u(n - 5) + u(n - 6)$

$h(n) = u(n + 2) - u(n - 3)$

(b) $x_1(n) = (..., 1, 1, 1, 1, ...)$ and $x_2(n) = (..., 1, 1, 1, 1...), N = 4$

5. Prove that convolution in time domain is equal to multiplication in frequency domain. Compute N-point DFT of the sequence : 10

(a) $x(n) = \delta(n)$

(b) $x(n) = \delta(n - n_0), 0 \leq n \leq N - 1$

6. Explain is Decimation-in-time FFT algorithm, Decimation-in-frequency FFT algorithm. 10

7. Design a Butterworth digital filter for : 10

Pass-band gain required : 0.95

Frequency up to which pass-band gain must remain steady, $f_1 = 500$ Hz

Amount of attenuation : 0.20

Frequency from which attenuation must start, $f_2 = 3000$ Hz.

8. Define and plot the followings : 10

(a) Delta function

(b) Step function

(c) Ramp function

(d) Exponential function

(e) Sinusoidal function.