Total number of printed pages – 7 B. Tech

BSCM 2101/BS 1101

First Semester Examination – 2008

MATHEMATICS – I

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any ***five*** *from the rest.*

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

   (a) What do you mean by integrating factor of a differential equation ?

   (b) What do you mean by general solution and particular solution of a differential equation ? What is the practical significance of these two concepts ?

(c) What is the wronskian ? What role does it play in getting solution of a differential equation ?

(d) What does the convergence of a power series means ? Why is it important ?

(e) Using Rodrigue's formula find $P_2(x)$.

(f) What is the relationship between $J_n(x)$ and $J_{-n}(x)$?

(g) Is there exist any asymptote for the curve $x^2 + y^2 = 25$? If so, find one.

(h) Find the radius of curvature at the origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$.

(i) How can you say a real square matrix is orthogonal ?

OR

What is the unit step function ? Why is it important in Laplace transform of a function ?

(j) Explain the conditions for which a system of linear equations will possess more than one solution.

OR

What is the Laplace transform of the function $f(t) = t^2$ if $0 < t < 1$

$= 0$ otherwise

2. (a) Find an integrating factor and solve the following differential equation : 5

$$2xy\,dx + 3x^2\,dy = 0.$$

(b) Reduce the following differential equation into linear form and solve : 5

$$\frac{dy}{dx} + \frac{y}{3} = \frac{(1-2x)y^4}{3}.$$

3. (a) Solve the following differential equation :

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

where $y(0) = 1.5$ and $y'(0) = 1$. 5

(b) Using method of variation of parameter, solve the following differential equation: 5

$$\frac{d^2y}{dx^2} + 9y = \sec 3x.$$

4. (a) Show that the radius of curvature at any point of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent. 5

(b) Find all asymptote of the following curve : 5

$$x^2(x-y)^2 + a^2(x^2 - y^2) = a^2xy.$$

5. (a) Find a power series solution in powers of x of the following differential equation : 5

$$(1-x^2)\,y'' = 2xy.$$

(b) Show that $G(u, x) = (1 - 2ux + u^2)^{\frac{-1}{2}}$ is the generating polynomial for the Legendre polynomial $P_n(x)$. 5

6. (a) Show that $\int J_{v+1}(x)\,dx = \int J_{v-1}(x)\,dx - 2J_v(x)$ where $J_v(x)$ is the Bessel's function of the first kind of order v. 5

(b) Reduce the following differential equation in to Bessel's differential equation and find a general solution in terms of Bessel's function : 5

$$y'' + x^2y = 0.$$

7. (a) Solve the following system of linear equations by Gauss elimination : 5

$$4x - 3y + 3z = 21$$
$$-x + 2y - 5z = -21$$
$$3x - 6y + z = 7$$

(b) Show that the rank of a matrix a is equal to the maximum number of linearly independent columns of A. 5

OR

(a) Find the Laplace transform of the following function : 5

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases}$$

(b) Using convolution theorem, find the inverse Laplace transform of the following function : 5

$$F(s) = \frac{1}{s(s^2 + 4)}.$$

8. (a) Find the inverse of the following matrix by using Guass-Jordan elimination method : 5

$$\begin{bmatrix} 1 & 8 & -7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Find a basis of eigen vector and diagonalize the matrix. 5

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

OR

(a) Solve the following differential equation by using Laplace transformation : 5

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = e^{-3t} - e^{-5t}$$

where $y(0) = 0$ and $y'(0) = 0$.

(b) Using Laplace transform solve the following integral equation : 5

$$y(t) = 1 - \int_0^t (t - \tau)\, y(\tau)\, d\tau.$$