0701209239

Total number of printed pages – 7

B. Tech/B. Arch

BSCM 2102/SCM 2002

Second Semester Examination – 2008

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

Figures in the right hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) How many solutions does the following linear system have and why :

$$x + 2y = 2$$

$$2x + 4y = 1$$

P.T.O.

(b) If $\lambda = 0$ is an eigenvalue of the matrix A, then what one can say about $|A|$.

(c) Find the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}.$$

(d) If A is any 5×5 matrix, and a polynomial associated with the matrix A is defined by $|\lambda I - A| = \sum_{n=0}^{n} \alpha_i \lambda^i$, then what is the determinant of the matrix A.

(e) Find the period of the following periodic functions $f(x) = \sin(6x)$ and $g(x) = \cos(3x)$.

(f) Identify which of the functions are even in the specified period

$$f(x) = \begin{cases} x-2, & 1 \le x \le 2 \\ 2-x, & 2 \le x \le 3 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} 2-x, & 1 \le x \le 2 \\ 2-x, & 2 \le x \le 3 \end{cases}$$

(g) Change the order of the integration

$$\int_0^{\infty}\int_0^{x} e^{-xy} y \, dy \, dx$$

(h) Find the parameter α such that the vectors $\upsilon_1 = (1, 1, \alpha)$, $\upsilon_2 = (1, 0, \alpha)$ and $\upsilon_3 = (0, \alpha, 0)$ are linearly dependent.

(i) Find the dimension of the vector space

$$V = \{\upsilon = (\upsilon_1, \upsilon_2, \upsilon_2) | \upsilon_1 + \upsilon_2 - \upsilon_3 = 0\}.$$

(j) If the set of vectors $\upsilon_1 = (1, 1, 1)$, $\upsilon_2 = (1, 0, 1)$, and $\upsilon_3 = (0, 1, 0)$ are dependent, then find their dependency.

2. (a) Solve the following linear system by Gauss elimination method : 5

$$x + z = 1$$
$$2x + z = 0$$
$$x + y + z = 1$$

(b) Find the inverse of the following matrix by Gauss-Jordan elimination method : 5

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

3. (a) If A is any square matrix, then show that $\frac{A + A^T}{2}$ is symmetric and $\frac{A - A^T}{2}$ is skew-symmetric. 5

(b) Find the eigenvalues and the set of all linearly independent eigen-vectors of the following matrix : 5

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

4. (a) If $A = (a_{ij})$ is any $n \times n$ matrix with $a_{ij} = 0$ for $j \leq i$, then show that $A^n = 0$. 5

(b) Find the matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix containing eigen-values of the matrix A along the diagonal and $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. 5

5. (a) Show that $\frac{\pi}{4} = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2 - 1}$ by finding the Fourier series of the periodic function. 7

$$f(x) = \begin{cases} \sin(x), & 0 \leq x \leq \pi \\ 0, & \pi \leq x \leq 2\pi \end{cases}$$

(b) Find the half range cosine Fourier series of the periodic function. 3

$$f(x) = \begin{cases} x, & 0 \leq x \leq 2 \\ 4 - x, & 2 \leq x \leq 4 \end{cases}$$

6. (a) Evaluate the double integral : 5

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx.$$

(b) Find the directional derivative of the function $\phi(x, y) = x^2y^3 - 4y$ at the point $(2, -1)$ in the direction of the vector $\mathbf{v} = (2, 5)$. 5

7. (a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using multiple integral. 5

(b) Evaluate the line integral $\int_c x^4 dx + xy\, dy$ by using Green's theorem where c is the triangular curve consisting of the line segments from (0, 0) to (1, 0), from (1, 0) to (0, 1) and from (0, 1) to (0, 0). 5

8. (a) Show that $\mathbf{F}(x, y, z) = (y^2z^3, 2xyz^3, 3xy^2z^2)$ is a conservative vector field, and evaluate $\int_{(0,0,0)}^{(1,0,1)} (y^2z^3\, dx + 2xyz^3\, dy + 3xy^2z^2\, dz)$. 5

(b) Evaluate $\iint_S \mathbf{F}.d\mathbf{S}$ where $\mathbf{F}(x, y, z) = (xy, y^2 + e^{xz^2}, \sin(xy))$ where S is the surface of the region bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$ and $y + z = 2$. 5
