Total number of printed pages – 8

B. Tech

CPEC 5302

Sixth Semester Examination – 2008

DIGITAL SIGNAL PROCESSING

Full Marks – 70

Time : 3 Hours

Answer either from Set-A or Set-B,
but not from both.

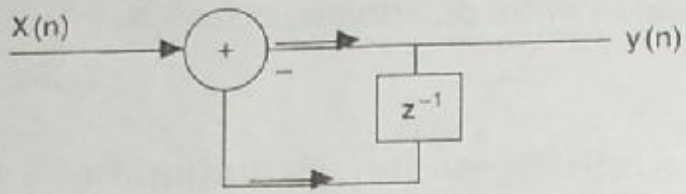
SET – A

Answer Question No. **1** *which is compulsory*
and any **five** *from the rest.*

The figures in the right-hand margin
indicate marks.

1. Answer the following questions : 2×10

   (a) Find the response of the system if a = 1, b = –1, x(n) = δ(n) and the system is initially at rest.



(b) Find out the Nyquist rate for the signal $x(t) = 25 \cos (500 \pi t)$.

(c) What is the stability condition of an LTI system ?

(d) At which band an ideal filter is distortionless ?

(e) How the DFT and DTFT of one discrete time signal related ?

(f) Find out the impulse response of the LTI system given by

$$y(n) = k_1 x(n) + k_2 x(n-1) + k_3 x(n-2).$$

(g) What are the advantages of FFT over DFT ?

(h) Draw the signal flow graph of a first order digital filter.

(i) Show whether the systems are (i) Linear / Non linear, (ii) TV/TIV.

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$y(n) = x(n^2).$$

(j) What is the aliasing effect ?

2. (a) Determine the impulse response for the given system described by difference equation. 6

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

(b) Compute and sketch the step response of the system. 4

$$y(n) = \frac{1}{M} \sum_{k=0}^{N-1} x(n-k).$$

3. (a) Determine convolution of the following pairs of signal by means of ZT. 6

$x_1(n) = 0.5^n u(n)$, $x_2(n) = \cos \pi n \, u(n)$.

(b) Consider the Fir filter represented as $y(n) = x(n) + x(n-4)$. Compute and sketch the magnitude and phase spectrum. 4

4. (a) Let x(n) be a real valued N point sequence. Develop a method to compute a N point DFT x'(k), which contains only the odd harmonics by using a real N/2 point DFT. 5

(b) Perform linear convolution of the following sequence by overlap add method. 5

$x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$

$h(n) = \{-1, 1\}$.

5. $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3)$

(i) Find the four point DFT of x(n). 5

(ii) If y(n) is the four point circular convolution of x(n) with itself, find y(n) and four point DFT Y(k). 5

6. Design an FIR digital filter approximating the ideal low frequency response.

$$H_d(\varpi) = \begin{cases} 1, & |\varpi| \le \frac{\pi}{6} \\ 0, & \frac{\pi}{6} \le |\varpi| \le \pi \end{cases}$$

(i) Determine the coefficients of 25 tap filter based on window method with a rectangular window. 5

(ii) Plot the magnitude and phase response of the filter. 5

7. (a) With impulse invariance, a first order pole in $H_a(s)$ at $s = s_k$ is mapped to a pole in H(Z) at $Z = e^{s_k T}$.

$$\frac{1}{s - s_k} \Rightarrow \frac{1}{1 - e^{s_k T} z^{-1}}$$

Determine how a second order pole is mapped with impulse invariance. 6

(b) A second order continuous time filter has a system function

$$H(s) = \frac{1}{s-a} + \frac{1}{s-b}.$$

Where $a < 0$ and $b < 0$ are real. Determine the locations of poles of H(Z) if the filter designed using impulse invariance technique with T = 2 sec. 4

8. (a) Find the direct form II realization for the system described by difference equation. 6

$$Y(n) = \frac{3}{4}y(n-1) - \frac{3}{4}y(n-2) + x(n) - \frac{1}{3}x(x-1)$$

(b) Explain the power spectrum estimation using the Bartlet method. 4

SET – B

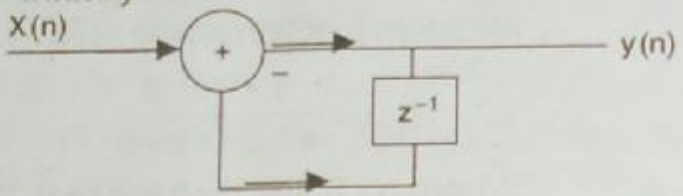
*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Find the response of the system if a = 1, b = −1, $x(n) = \delta(n)$ and the system is

initially at rest.



(b) Find out the Nyquist rate for the signal $x(t) = 25\ COS\ (500\ \pi t)$.

(c) What is the stability condition of an LTI system ?

(d) At which band an ideal filter is distortionless ?

(e) How the DFT and DTFT of one discrete time signal related ?

(f) Find out the impulse response of the LTI system given by

$y(n) = k_1x(n) + k_2x(n-1) + k_3x(n-2)$.

(g) What are the advantages of FFT over DFT ?

(h) Draw the signal flow graph of a first order digital filter.

(i) Show whether the systems are (i) Linear / Non linear, (ii) TV/TIV.

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$y(n) = x(n^2).$$

(j) What is the aliasing effect ?

2. (a) Determine the impulse response for the given system described by difference equation. 6

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

(b) Compute and sketch the step response of the system. 4

$$y(n) = \frac{1}{M}\sum_{k=0}^{N-1} x(n-k).$$

3. (a) Find the direct form II realization for the system described by difference equation. 6

$$Y(n) = \frac{3}{4}y(n-1) - \frac{3}{4}y(n-2) + x(n) - \frac{1}{3}x(x-1)$$

(b) Consider the Fir filter represented as $y(n) = x(n) + x(n-4)$. Compute and sketch the magnitude and phase spectrum. 4

4. (a) Let x(n) be a real valued N point sequence. Develop a method to compute a N point DFT x'(k), which contains only the odd harmonics by using a real N/2 point DFT. 5

(b) Perform linear convolution of the following sequence by overlap add method. 5

$x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$

$h(n) = \{-1, 1\}$.

5. $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3)$

   (i) Find the four point DFT of x(n). 5

   (ii) If y(n) is the four point circular convolution of x(n) with itself, find y(n) and four point DFT Y(k). 5

6. Determine the mean and the autocorrelation of the sequence x(n), which is the output of a ARMA (1, 1) process described by difference equation $x(n) = 0.5\ x(n-1) + w(n) - w(n-1)$. 10

7. For zero mean, jointly Gaussian random variable X1, X2, X3, X4 it is known that

   $E(X1\ X2\ X3\ X4) = E(X1\ X2)\ E(X3\ X4) + E(X1\ X3)\ E(X2\ X4) + E(X1\ X4)\ E(X2\ X3)$

   use this result to derive the mean square value of $r'_{xx}(m)$ and the variance which is

   $\text{Var}\,[r'_{xx}(m)] = E\,[|r'_{xx}(m)|^2] - E\,|[r'_{xx}(m)]|^2$ 10

8. Determine the coefficient {h(n)} of a linear phase FIR of length N = 15 which has a symmetric unit sample response and a frequency response that

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0, & k = 4, 5, 6, 7 \end{cases}$$

   satisfies the condition. 10