MCA-074
MCA-049

(9)

Total Pages—8

MCA
SCM 2005

First Semester Examination, 2003

DISCRETE MATHEMATICS

Full Marks : 70

Time : 3 hours

Answer Q. No. 1 which is compulsory and any **five** from the remaining questions

The figures in the right-hand margin indicate marks

1. (*i*) Let $f : A \rightarrow A$ be a function satisfying $f(f(x)) = x$. Is the function injective? surjective?

   (*ii*) Define an equivalence relation ~ on Z^+, the set of positive integers as follows: For $m, n \in Z^+$, $m \sim n \Longleftrightarrow m - n$ is divisible by 2. Write down the equivalence classes.

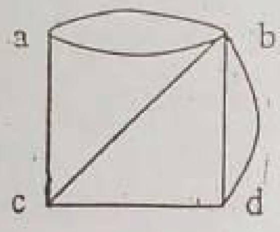
   (*iii*) State whether the statements are *true* or *false* :

   (*a*) If 256 is a perfect square, then 1024 is a perfect square.

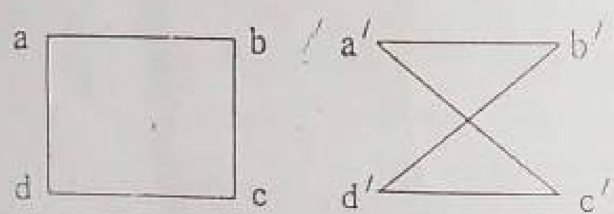
   (*b*) If 37 is a prime, then 148 is a prime.

(Turn Over)

(*iv*) Does the following graph have an Eulerian circuit ?

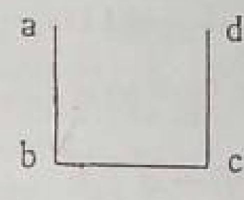


(*v*) Are the following graphs isomorphic ? Justify your answer.



(*Continued*)

(*vi*) Construct the complementary graph of the following graph and check if this graph is self complementary.



(*vii*) Define a binary operation o on N, the set of natural numbers as follows :

$$m \circ n = \text{minimum}\ (m, n)$$

Is the operation associative ?

(*viii*) $G = \{0, 1, 2, 3, 4, 5\}$ is a graph with respect to addition mod 6. Find all its proper subgroups.

(*ix*) Let Z^+ be the set of positive integers. Define a partial order $<$ on Z^+ by

$$m < n \Longleftrightarrow m \text{ divides } n.$$

Is this set a lattice ?

(*Turn Over*)

(*x*) Simplify the following Boolean expression :

$$x(x'+y)+y(y'+x') \qquad 2\times 10$$

2. (*a*) Find the number of integers between 1 and 1000 which are divisible by any of the integers 2, 3, 7 and 11. 5

(*b*) Given that $(p \to q)$ is *false*, determine the value of

$$(\bar{p} \vee \bar{q}) \to q \qquad 5$$

3. (*a*) Let $A = \{1, 2, 3, 4, 5\}$ 5

(*i*) Find the number of relations on A.

(*ii*) Find the number of reflexive relations on A.

(*iii*) How many of these relations are both reflexive and symmetric ?

(*b*) Let $A = \{1, 2, 3, 4\}$ and R is a relation on A defined by

$$R = \{(1, 2), (2, 3), (3, 2), (3, 4)\}$$

Find the transitive closure of A. 5

4. (*a*) Let $(P, \leq)$ be a partially ordered set. If the longest chain in P is of length n, then show that P can be partitioned into n disjoint antichains. 5

(*b*) Show by induction that :

$$(11)^{n+2} + (12)^{2n+1}$$

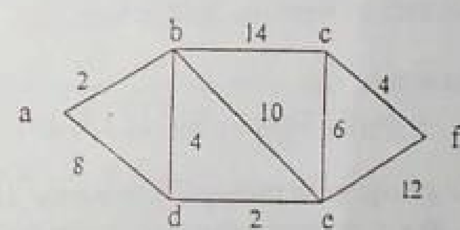
is divisible by 133. 5

5. (*a*) Define the degree of a vertex in a graph. Show that if $v_1, v_2, \ldots v_n$ are all the vertices of a graph, then

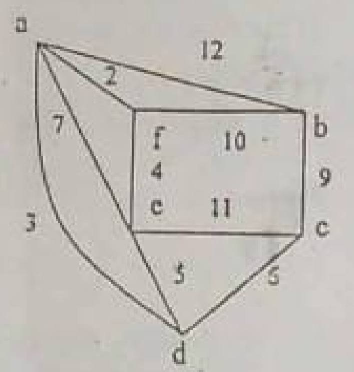
$$\sum_{i=1}^{n} d(v_i)$$

is even. Hence, show that there cannot be a graph with an odd number of vertices of odd degree. 5

(*b*) Use Dijkstra's algorithm to find the shortest distance from a to f in the following weighted graph. 5



6. (*a*) Determine a minimum spanning tree for the weighted graph given below : 5



(*b*) A binary tree has height h (that is the longest path has height h) (that is the longest path has length h). Find the maximum number of vertices in this tree. 5

7. (*a*) Show that

$$G = \{1, 2, 4, 7, 8, 11, 13, 14\}$$

is a group with multiplication mod 15 (that is, $m \star n$ = remainder when mn is divided by 15). 5

(*b*) Let A denote the set of all binary sequences of length n. Let $\oplus$ be a binary operation on A defined as follows. For $x, y \in A$, $x \oplus y$ is a sequence of length n, having 1 in those positions where x, y differ and having 0 in those positions where x, y are same. Let $w(x)$ be the number of 1's in x.

If $d(x, y) = w(x \oplus y)$ then show that $d(x, y) \leq d(x, z) + d(z, y)$ 5

8. (*a*) Let B be a lattice satisfying the following distributive property :

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

for all a, b, c in B show that B will satisfy

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

for all a, b, $c \in B$. 5

(*b*) Given the Boolean function f, write f in its conjunctive and disjunctive normal form : 5

x,	y,	z	$f(x, y, z)$
0	0	0	1
1	0	0	0
0	1	0	1
1	1	0	1
0	0	1	0
1	0	1	0
0	1	1	0
1	1	1	1