Fourth Semester Examination, April – 2005

QUANTITATIVE TECHNIQUES

Full Marks : 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any five from the rest.

The figures in the right-hand margin indicate marks for the questions.

1. Answer the following : 2×10

   (a) Define a slack variable and a basic variable.

   (b) Give a mathematical formulation of an assignment problem.

   (c) Define queue discipline.

   (d) Two cards are drawn from a pack of 52 cards; find the probability that both are kings.

P.T.O.

(e) A and B are events such that

$Pr(A \cup B) = 0.9$, $Pr(A) = 0.8$, $Pr(A \cap B) = 0.4$

Are A and B independent ?

(f) Let X be the number of heads obtained when a fair coin is tossed 400 times. Find the expected value and variance of X.

(g) A random variable X has the following density function :

$$f(x) = \begin{cases} kx & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

find the value of k.

(h) The moment generating function of a random variable X is $m_X(t) = e^{\lambda(e^t - 1)}$ find the first moment of X.

(i) If two independent random variables X and Y have mean 2 and 4 respectively, find the mean value of 2X + 3Y.

(j) The joint density of [X, Y] is given by

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y) & 0 \le x \le 2,\ 2 \le y \le 4 \\ 0, & otherwise \end{cases}$$

find the marginal distribution of X.

2. (a) Use the simplex method to solve the following LPP : 5

Maximize $Z = 3x_1 + 4x_2 + 4x_3$

Subject to :

$$3x_1 - x_3 \le 5$$

$$-9x_1 + 4x_2 + 3x_3 \le 12$$

$$-6x_1 + 2x_2 + 4x_3 \le 2$$

$$x_1, x_2, x_3 \le 0$$

(b) A company has six jobs to be processed by six machines. The following table gives the returns in rupees when the i-th job is assigned to the j-th machine. How should the job be assigned so asto maximize the overall return ? 5

	J_1	J_2	J_3	J_4	J_5	J_6
M_1	9	22	58	11	19	27
M_2	43	78	72	50	63	48
M_3	41	28	91	37	45	33
M_4	74	42	27	49	39	32
M_5	36	11	57	22	25	18
M_6	13	56	53	31	17	28

3. (a) Solve the following transportation problem whose cost matrix and demands are given below : 5

						Available :
	7	7	10	5	11	45
	4	3	8	6	13	90
	9	8	6	7	5	95
	12	13	10	6	3	75
	5	4	5	6	12	105
Demand :	120	80	50	75	85	

(b) Use cutting plane algorithm to solve the following LPP : 5

Maximize $= x_1 + 4x_2$

Subject to :

$x_1 + 6x_2 \leq 18$

$x_1 \leq 3$

x_1, x_2 are positive integers.

4. (a) A fast food shop has one drive-in window. Cars arrive at the rate of 2 cars every five minutes, following Poissonian distribution. The space in front of the window car accommodate 10 cars (other cars can wait outside the space, if necessary). The service time per customer is exponential with mean of 1.5 minutes. Derive the following : 5

(i) the probability that the facility is idle.

(ii) the expected number of customers writing to be served.

(iii) the expected waiting time until a customer reaches the window to place an order.

(iv) the probability that the waiting time will exceed the 10-space capacity.

(b) Given that Pr (A) = 0.35, Pr (B) = 0.73, $Pr(A \cap B) = 0.14$. 5

Find the values of

(i) $Pr(A / B)$

(ii) $Pr(\overline{A} \cup B)$

(iii) $Pr(\overline{A} \cap B)$

5. (a) There are 2 boxes; the first box contains 2 gol and 4 silver coins; the second box contains 4 g and 4 silver coins. If a coin, taken at random from one of the boxes is found to be gold, find the probability that it is taken from the first box. 5

(b) A student has to pass a minimum of 5 tests out of 10 tests conducted during the year to get promoted to the next class. If his chance of passing one test is 0.8, find the probability of his getting promoted. 5

6. (a) A new car, on average, has 3 defective parts. If you buy a new car, what is the probability that the car will have (i) no defective part (ii) atmost one defective part (iii) atleast 2 defective parts. 5

(b) A company produces X bulbs per day; if the mean value of X is 1000 and the variance of X is 100, find the value of the probability that

$900 < X < 1100$

you may use Chebyshev's inequality. 5

7. (a) The life of a tyre is a random variable X; if the mean value of X is 30,000 (kilometers) and the standard deviation is 5000 (kilometers), find the probability that a tyre will last for more than 40,000 kilometers. You may use the following table : 5

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-1/2t^2} \, dt$$

x	1.0	1.5	2.0	2.5	3.0
Φ (x)	0.3413	0.4332	0.4772	0.4938	0.4987

(b) Two independent random variables X and Y have mean 3 and 4 and variance of 4 and 1 respectively. Find the variance of $2X + 3Y$. 5

8. (a) A joint probability density function for two random variables X and Y is given by : 5

$$F(x, y) = \begin{cases} ke^{-3(x+y)}, x, y > 0 \\ 0 \quad , otherwise \end{cases}$$

find the value of k, also find the probability that $0 < X < 1$ and $0 < Y < 1$:

(b) Given the values of X and Y, calculate the correlation coefficient r. 5

X	2	3	4	5	6	7	8
Y	8	10	12	14	16	18	20