

Fourth Semester Examination – 2006

QUANTITATIVE TECHNIQUES - I

Full Marks : 70

Time : 3 Hours

Answer question No. 1 which is compulsory and any five questions from the remaining questions.

The figures in the right-hand margin indicate marks for the questions.

- IWL
1. Answer the following questions : 2×10
- (a) Explain the terms 'unbounded solution' and infeasible solution in relation to linear programming.
- (b) Find the starting basic solution using the north-east corner rule :

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3. (a) Joseph needs to assign four jobs to four workers ; the cost of performing a job is a function of the skills of the workers. The table given below summarizes the cost of the assignments. Moreover, worker 1 cannot do job 3 and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method : 5

	1	2	3	4
1	\$ 50	\$ 50	-	\$ 20
2	\$ 70	\$ 40	\$ 20	\$ 30
3	\$ 90	\$ 30	\$ 50	-
4	\$ 70	\$ 20	\$ 60	\$ 70

- (b) Solve the following transportation problem using Vogel's approximation method 5

	1	2	3	4	
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
	5	15	15	15	

4. (a) Maximize $z = 5x_1 + 7x_2$

subject to :

$$2x_1 + x_2 \leq 13$$

$$5x_1 + 9x_2 \leq 41$$

x_1, x_2 are positive integers,

using Branch and bound methods. 5

- (b) Visitors' parking in a college parking lot is limited to 5 spaces only. Cars, making use of this space arrive according to a Poissonian distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors, who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. Other cars who cannot find a parking space or a temporary space must go elsewhere.

Determine L_q and W_s . 5

5. (a) The following table gives an optimal solution of the following LPP :

$$\text{Maximize } z = 3x_1 + 5x_2$$

subject to :

$$3x_1 + 2x_2 \leq 18$$

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Basic variables	C_B	x_B	x_1	x_2	x_3	x_4	x_5
x_1	3	2	1	0	y_3	0	$-2/3$
x_4	0	0	0	0	$-2/3$	1	$4/3$
x_2	5	6	0	1	0	0	1
Non basic variables $x_3 = x_5 = 0$	$z = 36$		0	0	1	0	3

Discuss the effect on optimality of the solution if the object function is changed to $z = 3x_1 + x_2$. 5

- (b) A person passes a test with probability 0.6. If he appears in five tests, what is the probability of his passing three consecutive tests? 5

6. (a) You have 10 socks of five different colours. If you pick four of them at random, what is the probability that there is at least one right pair? 5
- (b) State and prove Baye's formula. 5
7. (a) Define a random variable. If X is a continuous random variable with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Find its moment generating function. 5

- (b) The average life of a bulb is 1000 hours and the standard deviation is 300 hours. Assuming normal distribution, find the probability
- (i) that a randomly picked bulb will last more than 1400 hours
- (ii) that a randomly picked bulb will last less than 500 hours 5

You may use the following table :

$$A(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$$

x	1.0	1.33	1.66	2.0
A(x)	.3413	.4082	.4515	.4742

8. (a) The joint density of $[X, Y]$ is given by

$$f(x, y) = \begin{cases} \frac{1}{8}(6-x-y) & 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0 & \text{Otherwise} \end{cases}$$

Find $E(X)$, $E(Y)$, $E(X|y)$ and $E(Y|x)$. 5

(b) Find the correlation coefficient ρ from the following table : 5

x	12	13	14	15	16	17	18
y	59.5	65.8	60.8	62.9	70.1	69	80

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Handwritten calculations:

$$1 - \frac{8}{3}$$

$$-\frac{2}{8} \cdot \frac{2}{10}$$

$$-\frac{2}{8} \cdot \frac{8}{3}$$

$$-\frac{6}{5} - \frac{1}{5}$$

$$\frac{3}{5} - \frac{4}{15}$$

$$-\frac{20}{3}$$

$$\frac{8}{15}$$

$$-C$$