Total number of printed pages – 6

MCA
SCM 2005

First Semester Examination – 2007

DISCRETE MATHEMATICS

Full Marks – 70

Time – 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

   (i) Let A, B be two sets such that A – B = B. Then what can you say about A and B ?

   (ii) Define countable set. Is the set of rational numbers countable ?

P.T.O.

(iii) Can a relation be both symmetric and antisymmetric ? If so, give an example.

(iv) What do you mean by complement of an element in a lattice ? Give an example of a complemented lattice.

(v) Is the transitive closure of an antisymmetric relation always antisymmetric ? Give an example.

(vi) Define chain and antichain with examples.

(vii) What are the atoms of the Boolean lattice P(S), where S = {a, b, c} ?

(viii) How can you say that two graphs are isomorphic ? Give examples of two graphs which are isomorphic.

(ix) What do you mean by cut-set ? How a cut-set is related to a spanning tree of a graph ?

(x) Define a cyclic group. Give an example of a cyclic group. Does a cyclic group always satisfy commutative law ?

2. (a) Let R be a reflexive relation on a set A. Show that R is an equivalence relation if and only if (a, b) and (a, c) are in R implies that (b, c) is in R. 5

(b) Using mathematical induction show that

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$$

5

3. (a) Let (P, ≤) be a partially ordered set. Suppose the length of the longest chains in P is n. Then show that the elements of P can be partitioned into n disjoint antichains. 5

(b) State the different principles of drawing a Hasse diagram for a partial ordering

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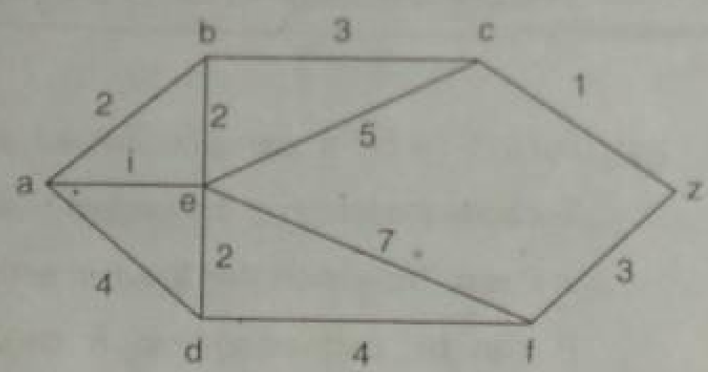
3. (a) Let (P, ≤) be a partially ordered set. Suppose the length of the longest chains in P is n. Then show that the elements of P can be partitioned into n disjoint antichains. 5

(b) State the different principles of drawing a Hasse diagram for a partial ordering relation. Draw the Hasse diagram for the relation R on the set of all positive divisors of 30, where a R b if a divides b. 5

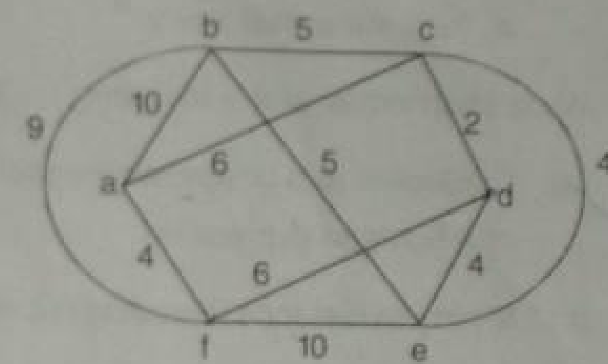
4. (a) In a graph with n vertices, if there exit a path from vertex v_1 to vertex v_2, then show that there is a path of no more than n – 1 edges from vertex v_1 to vertex v_2. 5

(b) Find the shortest path from the vertex a to vertex z for the following weighted graph. 5



5. (a) Show that in a graph every circuit has an even number of edges in common with every cut-set. 5

(b) Find a minimum spanning tree for the following graph stating the algorithm. 5



6. (a) Let (g, *) be a group and H be a non-empty subset of G. Show that (H, *) is a subgroup if for a and b in H, $a * b^{-1}$ is also in H. 5

(b) Let ({a, b}, *) be a semigroup where a * a = b. Then show that 5

(a) a * b = b * a

(b) b * b = b.

7. (a) State and prove Lagrange theorem. 5

(b) Let (A, $\le$) be a distributive lattice. If $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a, then show that $x = y$. 5

8. Write short notes of the following : 2.5×4

(a) Conjuctive and disjunctive normal forms of a Boolean expression

(b) Minimum distance decoding criterion

(c) Principle of duality for lattices

(d) Different methods of proofs.