

Fourth Semester Examination – 2007

QUANTITATIVE TECHNIQUES - I

Full Marks – 70

Time – 3 Hours

*Answer Question No. 1 which is compulsory  
and any five from the rest.*

*The figures in the right-hand margin  
indicate marks.*

1. Answer the following questions : 2x10

(a) Define the following terms :

Basic solution, Surplus variable, Pseudo-  
optimal solution.

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- (b) Give a mathematical formulation of the traveling salesman problem.
- (c) Write Little's formula.
- (d) Using North-West corner method, find the starting basic solution of the transportation problem.

				Supply	
	10	2	20	11	15
	12	7	9	20	25
	4	14	16	18	10
Demand	5	15	15	15	

- (e) What do you mean by queue-discipline ?
- (f) Two red and two black cards are lying face down on a table. If you guess their colours, what is the probability that your guess is right ?
- (g) A person has 0.8 probability of passing a test. If the person takes six tests, what

is the probability of his (her) passing at least four test ?

- (h) You purchase a lottery ticket worth 20 rupees. If the probability of your winning the first prize of one million rupees is one in one million, what is your expected profit ? [Assume that there is just one prize in this lottery]
- (i) A fair coin is tossed 100 times. If  $X$  is the number of heads obtained, find the expected value and variance of  $X$ .
- (j) A book has 2 misprints per page on average. If you open the book at random, what is the probability that there are more than 2 misprints on that page ?

2. (a) Solve graphically :

$$\text{Maximize } z = 5x + 4y$$

$$\text{subject to : } 6x + 4y \leq 24$$

$$x + 2y \leq 6$$

$$-x + y \leq 1$$

$$y \leq 2$$

$$x, y \geq 0$$

(b) Solve :

$$\text{Minimize } z = 4x + y$$

$$\text{subject to : } 3x + y = 2$$

$$4x + 3y \geq 6$$

$$x + 2y \leq 4$$

$$x, y \geq 0$$

3. (a) Find the starting solution of the transportation problem given below by Vogel's approximation method :

	Supply				
	10	2	20	11	15
	12	7	9	20	25
	14	14	16	18	10
Demand	5	15	15	15	

(b) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution.

Find - 5

(i) The average number of births per year

(ii) The probability that no births will occur in any one day.

4. Use branch and bound method to solve the following : 10

$$\text{Maximize } z = 5x + 7y$$

$$\text{subject to : } 2x + y \leq 13$$

$$5x + 9y \leq 41$$

$$x, y \geq 0 \text{ are integers.}$$

5. (a) Write short notes on : 5

Finite population models.

(c) The coefficients of a linear equation  $ax + b = 0$  are determined by throwing a die twice. Find the probability that the equation has an integral root. 5

6. (a) A bag contains 100 tokens, numbered 1 through 100. If you pick up one token at random, what is the probability that the number on the token is divisible by 2, 3 or 5 but not by all these three numbers? 5

(b) The mean and the variance of the mathematics marks of an examination are 46 and 15 respectively. Assuming that the marks are normally distributed: 5

(i) find the probability that a randomly picked person has scored more than 70

(ii) find the probability that a randomly picked person has scored less than 30.

7. (a) Define the moment generating function of a random variable  $X$ . Find the moment generating function of  $X$  if - 5

$$P_r(X=r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

(b) Let  $x_1, x_2, \dots, x_n$  be independent random variables with same mean  $\mu$  and same variance  $\sigma^2$ . find the mean and variance of - 5

$$Y = \frac{x_1 + x_2 + \dots + x_n - n\mu}{\sqrt{n}\sigma}$$

8. (a) The joint p.d.f. of two random variables  $X$  and  $Y$ , is given by

$$f(x,y) = \begin{cases} 6e^{-2x-3y}, & x > 0, y > 0 \\ 0 & \text{elsewhere;} \end{cases}$$

Find the following probabilities. 5 C

(i)  $P, (1 \leq X \leq 2, 2 \leq Y \leq 3)$

(ii)  $P, (X \geq 2, Y \geq 2)$

(b) Determine the marginal densities. Are X and Y independent? 5

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