Total number of printed pages – 6

MCA
SCM 2005

First Semester Examination – 2008

DISCRETE MATHEMATICS

Full Marks – 70

Time – 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right hand margin indicate full marks for the questions.

1. Answer the following questions : 2×10

   (i) Is the following statement a tautology ? Justify your answer.

   $$p \Rightarrow (q \Rightarrow p)$$

   (ii) Let A = {a, b, c, d}. The relation R = {(a, a), (a, b), (b, c), (c, d)} is defined on A. Find R^2.

P.T.O.

(iii) If {{1, 3, 5}, {2, 4}} is a partition of the set A = {1, 2, 3, 4, 5}, determine the corresponding equivalence relation R.

(iv) What do you mean by symmetric closure of a relation ? How can you find the symmetric closure of a relation R defined on the set A ?

(v) Can you find a cycle of length greater than one in a diagraph representing a partial order relation ? Justify your answer.

(vi) For any two elements a, b of a lattice, show that $a \vee b = b$ if $a \leq b$.

(vii) Is there any Boolean algebra having nine elements ? Justify your answer.

(viii) Write the algorithm of preorder and post order searching of a tree.

(ix) How can you know whether an undirected graph possesses an Euler path or not ?

(x) What do you mean by minimum distance of an encoding function ? How can you find the minimum distance of a group code ?

2. (a) Prove by mathematical induction that n divides $n^3 - n$ for every positive integer n. 5

(b) Find an explicit formula for the sequence defined by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$ and $C_2 = 3$. 5

3. (a) If R is a relation defined on $A = \{a_1, a_2, a_3, ..., a_n\}$, then show that $M_{R^2} = M_R \otimes M_R$. 5

(b) State Warshall Algorithm to find the transitive closure of a relation. Hence, find the transitive closure of a relation R defined on the set {a, b, c, d} whose matrix representation is given below. 5

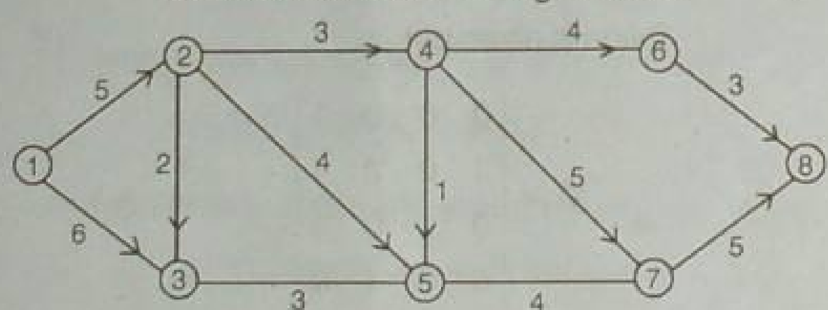
$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4. (a) Let L be a bounded lattice with at least two elements. Show that no element of L is its own complement. 5

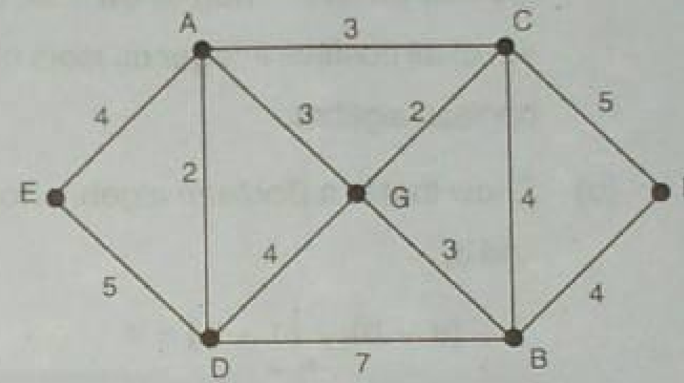
(b) Let A be a finite nonempty poset with partial order $\leq$. Show that A has at least one maximal element and at least one minimal element. 5

5. (a) Let the number of edges of a graph G be m and number of vertices be n. Then show that G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$. 5

(b) Find a maximum flow in the given network where each edge is labelled with its maximum capacity ; node 1 being the source and node 8 being the sink. 5



6. (a) Write the Kruskal's algorithm to find a minimal spanning tree for a graph. Using the algorithm, find a minimal spanning tree for the following graph. List the edges in the order in which they are chosen. 5



(b) Let R be a symmetric relation on a set A. Then prove that the following statements are equivalent. 5

(i) R is an undirected tree

(ii) R is connected and acyclic.

7. (a) Show that the intersection of two congruence relation on a semigroup is a congruence relation. 5

(b) Let G be an abelian group with identity element e and $H = \{ x : x^2 = e \}$. Show that H is a sub group of G. 5

8. (a) Let $n = p_1 p_2 p_3 \ldots p_k$, where the p_i are distinct primes. Then show that D_n, the set of all positive integer divisors of n, is a boolean algebra. 5

(b) Show that in a Boolean algebra, for any a and b,

$$(a \vee b) \vee (a \wedge b') = a$$ 5